CHANGE IN SHOCK-WAVE PRESSURE IMPULSE WITH DISTANCE NEAR THE SOURCE

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One of the quantities that characterizes the destructive action of an explosive source is the shock-wave pressure impulse I_s, which is defined as the integral over time of the pressure disturbance in the compression phase of the explosion wave [1]. Change in shock-wave impulse affects the behavior of the low frequency portion of the spectrum, which is also of importance [2]. Analysis of experimental data [1-6] has shown that near the charge the impulse $I_s(r)$ falls off with distance r more slowly than follows from conventional geometric divergence [7]. Moreover, numerical calculations [8], which have been confirmed experimentally [6], have shown that the initial abrupt falloff of impulse with distance is replaced by an increase in $I_s(r)$ for a certain r range, after which further slow reduction of impulse occurs, tending only asymptotically to a law $I_s \sim 1/r$. The existence of a relative maximum in the function $I_s(r)$ and its slow falloff as compared to $1/r$ were explained in [6] as due to transfer of impulse to the medium by detonation products which liberate shock wave energy over some time interval. In [3] the slow decay of $I_s(r)$ was explained by nonlinear effects on the shock wave front, i.e., the existence and evolution of a shock front. In [3] the peak approximation of Kirkwood–Beatty theory was used, which does not consider correct pressure distribution in the tail portion of the shock wave. Moreover, in calculating $I_s(r)$ the nonlinear transformation of shock-wave profile was not considered (quasiexponential approximation). However, as was shown in [9], correct calculation of nonlinear effects in the Kirkwood–Beatty approximation leads to a dependence $I_s(r) \sim 1/r$, which does not agree with the experimental data of [1-6].

This lack of clarity in interpreting the slow decay of shock wave pressure impulse with distance as compared to the function $I_s(r) \sim 1/r$ has stimulated the present study, the goal of which is to clarify the influence of nonlinear effects on the shock-wave front and flow of the medium upon the function $I_s(r)$.

1. We will consider the spherically symmetric motion of a continuous medium:

$$v_t + vv_r + \frac{1}{\rho} p_r = 0, \quad \rho_t + (\rho v)_r + \frac{2}{r}(\rho v) = 0, \tag{1.1}$$

where v is velocity; p, pressure; ρ, density. Combining the equations of (1.1), we obtain a relationship expressing the conservation of impulse:

$$p'_r = -\left\{(\rho v)_t + (\rho v^2)_r + \frac{2}{r}(\rho v^2)\right\}. \tag{1.2}$$

We will introduce the following notation: $T_1(r)$, time of arrival of the shock front at a point at distance r; $T_2(r)$, time of arrival of that portion of the wave where the pressure perturbation p' in the wave is equal to zero $p'(T_2) = 0$; at $t > T_2$ the pressure perturbation changes to the opposite sign, corresponding to the rarefaction phase. In the future, the subscript 1 placed on hydrodynamic parameters will indicate their values on the shock front, while 2 will indicate values at $t = T_2(r)$. Integrating Eq. (1.2) over the time interval $T_1 \leqslant t \leqslant T_2$ at constant r, we obtain:

$$\frac{dI_s}{dr} = -\left\{\frac{dE_s}{dr} + \frac{2}{r}E_s\right\} - \left\{\rho_2 v_2 - \frac{dT_2}{dr}\rho_2 v_2^2\right\} + \left\{\rho_1 v_1 - \frac{dT_1}{dr}\left(\rho_1 v_1^2 + p'_1\right)\right\}, \tag{1.3}$$

where $I_s = \int\limits_{T_1}^{T_2} p'dt$ is the pressure impulse; $E_s = \int\limits_{T_1}^{T_2} \rho v^2 dt$ is the dynamic impulse.

If we consider the Rankin–Hugoniot relationship on the shock front [10], it is simple to prove that the identity

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$$\rho_1 v_1 - \frac{dT_1}{dr}(p_1' + \rho_1 v_1^2) \equiv 0 \qquad (1.4)$$

is satisfied. It is evident from Eqs. (1.3), (1.4) that the shock-wave pressure impulse is dependent not only on the values of the dynamic impulse, but also on the values of the hydrodynamic quantities on the boundary between the compression and rarefaction phases at $t = T_2(r)$, the position of which in time for a given r is defined by the flow behind the shock-wave front:

$$\frac{dI_s}{dr} = -\left\{\frac{dE_s}{dr} + \frac{2}{r}E_s\right\} - \left\{\rho_2 v_2 - \frac{dT_2}{dr}\rho_2 v_2^2\right\}. \qquad (1.5)$$

In order to determine which of the two terms in Eq. (1.5) is the dominant one in controlling the behavior of $I_s(r)$, we will perform further simplifications. Since empirical laws describe the behavior of shock-wave parameters well in the distance range $r \geqslant r_0 \approx 10R_0$, where the ratios $p_1'n/\rho_0 c_0^2$ and v_1/c_0 are sufficiently small compared to unity (see, for example, [3]), in transformations of Eq. (1.5) we will consider only quadratically nonlinear terms. Here ρ_0 is the equilibrium density of the medium; c_0, speed of sound; n, adiabatic index; R_0, radius of source.

In the linear approximation, from Eq. (1.1) and the Rankin–Hugoniot relationships on the shock front we have the simple dependence

$$v = p'/\rho_0 c_0 + \int_{T_1}^{t} p'(t)\,dt/\rho_0 r. \qquad (1.6)$$

Using Eq. (1.6), for E_s we obtain

$$E_s = \frac{1}{\rho_0 c_0^2}\int_{T_1}^{T_2}(p')^2\,dt + \frac{1}{\rho_0 r^2}\int_{T_1}^{T_2}\left[\int_{T_1}^{t} p'dt\right]^2 dt + \frac{I_s^2}{\rho_0 c_0 r}. \qquad (1.7)$$

With consideration of Eq. (1.6) the second term in Eq. (1.5) reduces to the form

$$\rho_2 v_2 - \frac{dT_2}{dr}\rho_2 v_2^2 \approx \frac{I_s}{r}\left(1 - \frac{I_s}{\rho_0 c_0 r}\right), \qquad (1.8)$$

since $dT_2/dr \approx 1/c_0$. From Eqs. (1.5), (1.7), (1.8) we finally obtain an equation for I_s

$$\frac{dI_s}{dr} + \frac{I_s}{r}\left(1 + \frac{2}{\rho_0 c_0}\frac{dI_s}{dr}\right) = -\left\{\frac{dE_s'}{dr} + \frac{2}{r}E_s'\right\}, \qquad (1.9)$$

where $E_s' = \frac{1}{\rho_0 c_0^2}\int_{T_1}^{T_2}(p')^2\,dt$. If we integrate over time between limits $T_1 \leqslant t < \infty$, we can obtain an equation for the total pressure impulse of the explosive wave $I = \int_{T_1}^{\infty} p'dt$:

$$\frac{dI}{dr} + \frac{I}{\rho_0 c_0 r}\left(\frac{I}{r} + 2\frac{dI}{dr}\right) = -\left\{\frac{dE}{dr} + \frac{2}{r}E\right\}, \qquad (1.10)$$

where

$$E = \frac{1}{\rho_0 c_0^2}\int_{T_1}^{\infty}(p')^2 dt.$$

2. We will first consider Eq. (1.9) for shock-wave impulse in water. Energy dissipation on the shock front causes the quantity $E_s' = E_s'(r_0)\xi^{-(2+\varkappa)}$ to fall more rapidly ($\varkappa > 0$) than in a linear wave, where $\varkappa = 0$; $\xi = r/r_0$. In solving Eq. (1.9) we can use a theoretical dedependence for E_s' [11, 12] which agrees well with experimental data, however it is more convenient to use an empirical law for E_s' [2, 3, 5], where $\varkappa = 0.08$–0.12.

We will introduce the dimensionless quantity $S = I_s(r)\xi/I_s(r_0)$, characterizing the deviation of the law $I_s(r)$ from spherical, for which $S \equiv 1$. Equation (1.9) for S is written in the form

$$\frac{dS}{d\xi} = \frac{\alpha\xi^{-\varkappa} + \beta S^2/\xi}{\xi^2 + \beta S}. \qquad (2.1)$$

Here the parameters $\alpha = \varkappa E_s'(r_0)/I_s(r_0)$ and $\beta = 2I_s(r_0)/\rho_0 c_0 r_0$, dependent on r_0, are defined from empirical laws for E_s' and I_s, referenced to a TNT charge of mass Q (kg) and density $\rho_* = 1.6 \cdot 10^3$ kg/m^3 [3, 5]:

$$\alpha = 0.014(r_0/Q^{1/3})^{-1.13}, \ \beta = 0.049(r_0/Q^{1/3})^{-1.91}.$$

It is simple to obtain a solution from Eq. (2.1) corresponding to the limiting condition $\xi = 1$, $S = 1$:

$$S = \frac{\xi^2}{\beta}\left\{\left[1 + \frac{1}{\xi^2}\left(\beta(2+\beta) + \frac{2\alpha\beta}{1+\varkappa}\left(1 - \xi^{-(1+\varkappa)}\right)\right)\right]^{1/2} - 1\right\}. \quad (2.2)$$

From Eq. (2.2) at significant distances from the explosion point $\xi \gg 1$ we find that the values of S increase and tend to a definite constant value:

$$S \approx 1 + \frac{\beta}{2} + \frac{\alpha}{1+\varkappa}\left(1 - \xi^{-(1+\varkappa)}\right), \quad (2.3)$$

which agrees with the experimental data of [5], where the law of change of shock-wave impulse in water is represented by an expression of the following form:

$$I_s = \rho_0 c_0 R_0 \begin{cases} \dfrac{0.75}{R^{0.92}}, & R < 100, \\ \dfrac{1.12}{R}, & R > 100, \end{cases}$$

$R = \xi A^{-1/2} r_0/R_0$ is the reduced distance, $0.6 \leqslant A \leqslant 4.5$. Figure 1 shows: 1, theoretical [Eq. (2.2)] and 2, experimental [5] curves of S as a function of reduced distance R; a, A = 0.6; b, A = 4.6. The dashed lines indicate the possible behavior of a theoretical dependence which would correspond to smooth change of S near R = 100.

It should be noted that in Eq. (2.3) the term proportional to β considers the contribution of liquid flow to shock-wave impulse, since its appearance is due to consideration of the second term on the right side of Eq. (1.6), which does not disappear as $c_0 \to \infty$. The term proportional to α considers nonlinear energy dissipation effects in the shock front, since in a linear wave $\varkappa = 0$ and $\alpha = 0$. Since for $r_0/Q^{1/3} \approx 0.52 < 1$ the ratio of the second term of Eq. (2.3) to the first term $\beta(1+\varkappa)/2\alpha = 2.7(r_0/Q^{1/3})^{-0.78}$ is significantly greater than unity, it can be assumed that the increase in S is caused mainly by the existence of liquid flow behind the shock-wave fronts. Beginning at a distance $r_0 > 2.7Q^{1/3}$, where $\beta(1+\varkappa)/2\alpha < 1$, the increase in S is caused by nonlinear energy dissipation effects in the shock front. However, at such distance the increment in S is 2-3 orders of magnitude smaller than unity, therefore the change in S does not exceed tenths of a percent, i.e., will be practically indetectable. Consequently, the slow falloff of shock-wave pressure impulse (increase in S) with distance can be explained by the effect of liquid motion on the flow behind the shock-wave front.

3. We will now consider Eq. (1.10) for total pressure impulse of an explosive wave, which with the substitution $W = I(r)/I(r_0)$ reduces to the form

$$\frac{dW}{d\xi} = \frac{\alpha_1 \xi^{-(3+\varkappa)} - \dfrac{\beta_1 W^2}{2\xi^2}}{1 + \beta_1 W/\xi}, \quad (3.1)$$

where $\alpha_1 = \varkappa E(r_0)/I(r_0)$, $\beta_1 = 2I(r_0)/\rho_0 c_0 r_0$. If we neglect the term proportional to α_1, then a solution of Eq. (3.1) satisfying the boundary conditions W = 1 at $\xi = 1$ can be obtained in the form

$$W = \left(\frac{\frac{3}{2}\beta_1 W/\xi + 1}{\frac{3}{2}\beta_1 + 1}\right)^{1/3},$$

whence it follows that the flow of the medium leads to reduction in total exposion wave pressure impulse, since at $\beta_1 > 0$ for $\xi \to \infty$ $W \to \left(\frac{3}{2}\beta_1 + 1\right)^{-1/3} < 1$. Since it follows from Eq. (1.6) that as $t \to \infty$ the total impulse vanishes, i.e., $I(r_0) \approx 0$, then, neglecting the terms in Eq. (3.1) proportional to β_1 ($\beta_1 < \alpha_1$), we obtain the simple solution:

$$W = \frac{\alpha_1}{2+\varkappa}\left(1 - \xi^{-(2+\varkappa)}\right). \quad (3.2)$$

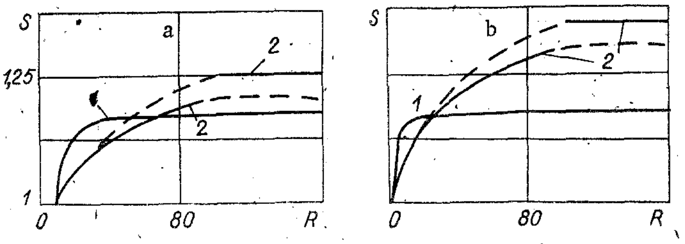

Fig. 1

As follows from Eq. (3.2), nonlinear energy dissipation effects on the shock front lead to an increase in total explosion wave pressure impulse.

In conclusion, we note that the slow falloff of shock-wave pressure impulse with distance, as compared to geometric divergence, which occurs upon detonation of a spherical charge in liquid has also been observed for an explosion in air (5). Since such behavior of the impulse is caused by the existence of a flow of the medium behind the shock front, it is evident that with increase in mass of the detonation products and with increase in their expansion rate the effect of retarded falloff of shock-wave pressure impulse will appear more markedly.

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